Junior Kangaroo 2019

Solutions

1. C The only digit 4 is at the end of the number and hence to obtain a number which reads the same from left to right as it does from right to left (known as a palindromic number), the first step is to erase the 4 leaving the number 1 232 331. There are now three different possibilities to produce a palindromic number - erasing the two 2s to leave 13 331, erasing the final two 3s to obtain 12 321 or erasing the first 2 and the either the last or second last 3 to obtain 13 231. However, in each case three digits have been erased.

2. C From the diagram, it can be seen that the sides of the larger squares are $2 \times 20\,\text{cm} = 40\,\text{cm}$ and $3 \times 20\,\text{cm} = 60\,\text{cm}$. Therefore the distance Adam walks is $(5 \times 20 + 5 \times 40 + 2 \times 60)\,\text{cm} = 420\,\text{cm}$.

3. D The river is 120 m wide and represents $(1 - \frac{1}{4} - \frac{1}{3}) = \frac{5}{12}$ of the length of the bridge. Therefore $\frac{1}{12}$ of the length of the bridge is 24 m. Hence the total length of the bridge is $12 \times 24\,\text{m} = 288\,\text{m}$.

4. D Let Evie's age in years now be x. The information in the question tells us that $x + 4 = 3(x - 2)$. Therefore $x + 4 = 3x - 6$ and hence $2x = 10$ and $x = 5$. Therefore in one year's time Evie will be 6.

5. E The areas of the two rectangles are $(8 \times 10)\,\text{cm}^2 = 80\,\text{cm}^2$ and $(9 \times 12)\,\text{cm}^2 = 108\,\text{cm}^2$. Since the area of the black region is $37\,\text{cm}^2$, the area of the unshaded region is $(80 - 37)\,\text{cm}^2 = 43\,\text{cm}^2$. Hence the area of the grey region is $(108 - 43)\,\text{cm}^2 = 65\,\text{cm}^2$.

6. B The information in the question tells us that both triangle SPQ and triangle QRS are right-angled. The area of the quadrilateral $PQRS$ is equal to the sum of the areas of triangle SPQ and triangle QRS.

Therefore the area of $PQRS$ is $\frac{1}{2}(11 \times 3)\,\text{cm}^2 + \frac{1}{2}(7 \times 9)\,\text{cm}^2 = \frac{1}{2}(33 + 63)\,\text{cm}^2 = \frac{1}{2}(96)\,\text{cm}^2 = 48\,\text{cm}^2$.

7. E Let the number of pupils who like neither subject be x. Hence the number who like both subjects is $2x$. Therefore the number of pupils who like only Maths is $20 - 2x$ and the number who like only English is $18 - 2x$. Since there are 30 pupils in my class, we have $(20 - 2x) + 2x + (18 - 2x) + x = 30$ and hence $38 - x = 30$. This has solution $x = 8$ and hence the number of pupils who like only Maths is $20 - 2 \times 8 = 4$.

8. C Since the mean of the original five numbers is 25, their total is $25 \times 5 = 125$. The total of the new set of five numbers is $125 + 5 + 10 + 15 + 20 + 25 = 200$. Therefore the mean of the new set of five numbers is $200 \div 5 = 40$.

9. B Since the product of the two positive integers is 240, the possible pairs of integers are (1, 240), (2, 120), (3, 80), (4, 60), (5, 48), (6, 40), (8, 30), (10, 24), (12, 20) and (15, 16). The respective sums of these pairs are 241, 122, 83, 64, 53, 46, 38, 34, 32 and 31. Of these, the smallest value is 31.

10. D Initially there are $(39 - 23) = 16$ more boys than girls in the group. Each week $(8 - 6) = 2$ more girls than boys join the group. Therefore it will take $16 \div 2 = 8$ weeks for the number of girls to equal the number of boys. Hence the total number of people in the group when this occurs is $2 \times (39 + 8 \times 6) = 2 \times 87 = 174$.

11. A If N were divisible by 55, then it would also be divisible by 5 and 11, making three statements true. Hence N is not divisible by 55. Therefore exactly two of the remaining statements are true. It is not possible for N to be both less than 10 and divisible by 11, and it is not possible for N to be divisible by both 5 and 11 without also being divisible by 55. Therefore the two true statements are N is divisible by 5 and N is less than 10. Hence the value of N is 5.

12. C The perimeter of the square is $4 \times 9\,\text{cm} = 36\,\text{cm}$. Therefore, since the perimeter of the square and the equilateral triangle are the same, the side-length of the equilateral triangle is $36\,\text{cm} \div 3 = 12\,\text{cm}$. Hence the length of each of the longer sides of the rectangle is 12 cm. Since the perimeter of the rectangle is also 36 cm, the length of each of the shorter sides of the rectangle is $(36 - 2 \times 12)\,\text{cm} \div 2 = 6\,\text{cm}$.

13. C To fill the box, the side-length of the cube needs to divide exactly into the length, width and height of the box. To obtain the minimum number of cubes to fill the box, we need this side-length to be as large as possible. Therefore we need this side-length, in cm, to be the highest common factor of 30, 40 and 50, which is 10. Hence with cubes of side-length 10 cm, we get the minimum to fill the box. Therefore the minimum number of cubes required is $(30 \div 10) \times (40 \div 10) \times (50 \div 10) = 3 \times 4 \times 5 = 60$.

14. A In each week, the number of pages Henry reads is $25 + 6 \times 4 = 49$. Now note that $290 = 5 \times 49 + 45$ and that $45 = 49 - 4$. Therefore, since Henry reads 49 pages a week and 4 pages every day except Sunday and he starts reading the book on a Sunday, it will take him 5 weeks and 6 days to finish the book. Hence he will take 41 days to finish the book.

15. D Since $1 + 2 + 3 + 4 = 10$ and the sum of Amy's position, Bob's position and Dee's position is 6, Cat came fourth. Hence, since the sum of Bob's position and Cat's position is also 6, Bob came second. Therefore, since Bob finished ahead of Amy, Amy finished third. Therefore Dee came first in the tournament.

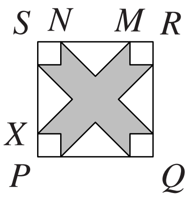
16. D Note first that the sum of the numbers on the eight cards is 36. Therefore the sum of the numbers on the cards in each of the boxes is 18. There are only three cards in box P and hence the possible combinations for the numbers on the cards in box P are $(8, 7, 3)$, $(8, 6, 4)$ and $(7, 6, 5)$ with the corresponding combinations for box Q being $(6, 5, 4, 2, 1)$, $(7, 5, 3, 2, 1)$ and $(8, 4, 3, 2, 1)$. The only statement which is true for all three possible combinations for box Q is that the card numbered 2 is in box Q. Hence the only statement which must be true is statement D.

17. A The interior angles of an equilateral triangle, a square and a regular pentagon are $180° \div 3 = 60°$, $2 \times 180° \div 4 = 90°$ and $3 \times 180° \div 5 = 108°$ respectively. Therefore the size of the obtuse $\angle UVW$ is $108° + 90° - 60° = 138°$. Since the pentagon and the square share a side and the square and the equilateral triangle also share a side, the side-length of the pentagon is equal to the side-length of the equilateral triangle. Therefore $UV = VW$ and hence the triangle UVW is isosceles and $\angle VWU = \angle WUV$. Therefore the size of $\angle WUV$ is $(180° - 138°) \div 2 = 21°$.

18. E Consider the left-hand column and the top row of the diagram. When we add the values in these lines together, we obtain $3\spadesuit + 3\blacklozenge = 105$ and hence $\spadesuit + \blacklozenge = 35$. Therefore, from the middle column, $35 + \clubsuit = 47$ and hence $\clubsuit = 12$. Therefore the value of $\spadesuit + \blacklozenge - \clubsuit$ is $35 - 12 = 23$.

19. C Since the non-shaded squares are congruent and since $MN = 6\,\text{cm}$, both SN and MR have length $(10 - 6)\,\text{cm} \div 2 = 2\,\text{cm}$. Therefore the areas of the four non-shaded squares are each $(2 \times 2)\,\text{cm}^2 = 4\,\text{cm}^2$.

Label the point X on SP as shown. Since all of the non-shaded squares are congruent, the lengths of SM and SX are equal and hence triangle MSX is an isosceles, right-angled triangle with angles of 90°, 45° and 45°. Therefore, since the non-shaded triangles are all isosceles and have an angle of 45°, they are also right-angled. Therefore these four triangles can be fitted together to form a square of side-length 6 cm. Hence the total area of the non-shaded triangles is $(6 \times 6)\,\text{cm}^2$. Therefore the area of the shaded region is $(10 \times 10 - 4 \times 4 - 6 \times 6)\,\text{cm}^2 = (100 - 16 - 36)\,\text{cm}^2 = 48\,\text{cm}^2$.



20. B Consider a 2×7 table with entries a and b in the first column. Since the entries in the following columns are the sum and the difference of the numbers in the previous column, the completed table will be as shown below.

a	$a + b$	$(a + b)$ $+ (a - b)$ $= 2a$	$2a + 2b$	$(2a + 2b)$ $+ (2a - 2b)$ $= 4a$	$4a + 4b$	$(4a + 4b)$ $+ (4a - 4b)$ $= 8a$
b	$a - b$	$(a + b)$ $- (a - b)$ $= 2b$	$2a - 2b$	$(2a + 2b)$ $- (2a - 2b)$ $= 4b$	$4a - 4b$	$(4a + 4b)$ $- (4a - 4b)$ $= 8b$

Since the numbers in the final column of Carl's table are 96 and 64, we have $8a = 96$ and $8b = 64$ which have solution $a = 12$ and $b = 8$. Therefore the sum of the numbers in the first column of Carl's table is $12 + 8 = 20$.

21. D Let the number of electric eels be x, the number of moray eels be y and the number of freshwater eels be z. The information on the notice tells us that $y + z = 12$, $x + z = 14$ and $x + y = 16$. When you add these three equations, you obtain $2x + 2y + 2z = 42$ and hence $x + y + z = 21$. Therefore the number of eels in the tank is 21.

22. A Let x km be the distance Geraint cycles and let t hours be the time his journey should take if he is to be on time. Since $\frac{\text{distance}}{\text{speed}} = \text{time}$, the information in the question tells us that $\frac{x}{15} = t + \frac{1}{6}$ and that $\frac{x}{30} = t - \frac{1}{6}$. When we subtract the second equation from the first, we obtain $\frac{x}{30} = \frac{2}{6}$ and so $x = 10$. Hence, from the second equation, $\frac{10}{30} = t - \frac{1}{6}$ and so $t = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$. Therefore, to arrive on time, Geraint needs to travel 10 km in $\frac{1}{2}$ hour, which is an average speed of 20 km/h.

23. A Since any two cells which share a vertex are coloured differently, the centre cell in the top row could only be coloured red or green. The cell below that cannot be coloured blue or yellow or the same colour as the centre cell in the top row and so is coloured green or red opposite to the choice of the colour to the first cell considered. The remaining cells in the second row can then be coloured out from the centre with only one possible colour for each cell. This argument can then be repeated for the colours of the third row and the fourth row, which turn out to be exactly the same as the colours of the first and second row respectively, as shown in the diagram. Hence the colour used for the cell marked X is red.

R	B	R/G	Y	G
G	Y	G/R	B	R
R	B	R/G	Y	G
G	Y	G/R	B	R

24. **B** Since the ratios of frogs to toads in the two ponds are $3 : 4$ and $5 : 6$ respectively, the numbers of frogs and toads are $3x$ and $4x$ in the first pond and $5y$ and $6y$ in the second pond for some positive integers x and y. Therefore, since there are 36 frogs in total, we have $3x + 5y = 36$. Since both 3 and 36 are multiples of 3, y is also a multiple of 3 and since $5y \leq 36$ we have $y = 3$ or 6. If y were 3, x would be 7 and the total number of toads would be $4 \times 7 + 6 \times 3 = 46$. Similarly, if y were 6, x would be 2 and the total number of toads would be $4 \times 2 + 6 \times 6 = 44$. Hence, the largest possible number of toads in the ponds would be 46.

25. **E** Each floor has 35 rooms. On every floor except floor 2, the digit 2 will be used for rooms 'n02', 'n12', 'n20' to 'n29' (including 'n22') and 'n32'. Hence the digit 2 will be used 14 times on each floor except floor 2. On floor 2, the digit 2 will be used an extra 35 times as the first digit of the room number. Therefore the total number of times the digit 2 will be used is $5 \times 14 + 35 = 105$.